II. *De Inventione Centri Oscillationis. Per* Brook Taylor *Armig. Regal. Societat. Sodal.*

Definitio.

Est Centrum Oscillationis punctum quoddam in corpore pendulo, cujus vibrationes singulæ eodem modo atq; eodem tempore peraguntur, ac si illud solum ad eandem distantiam a puncto suspensionis filo suspenderetur.

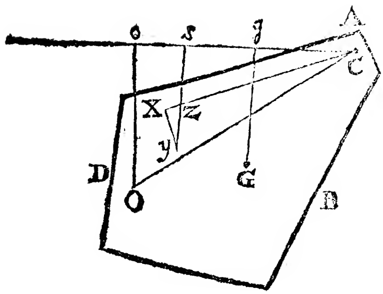
PER se vix satis manifestum est in corpore aliquo dari hujusmodi punctum: utpote cujus acceleratio debeat, (*per hanc def.*) in omnibus inclinationibus corporis penduli ad Horizontem, perinde esse, ac si a propriâ tantùm gravitate urgeatur; reliquis particulis totius corporis ejus motum proprium haud perturbantibus. Itaq; in ordine ad inventionem hujus Centri, præmittenda est una atq; altera propositio, unde constet tale punctum dari.

Prop. 1. Prob. 1

In corporis Oscillantis datâ quâvis inclinatione ad Horizontem, invenire punctum cujus acceleratio perinde sit, ac si ab ipsius propriâ tantum gravitate urgeatur.

Sit A B D corporis propositi sectio in plano ad Horizontem perpendiculari, in quo movetur centrum gravitatis G, centro suspensionis existente C. Distinguatur corpus in elementa prismatica plano A B D perpendi-

cularia,



cularia, adeoque Horizonti ſemper parallela; ut facile patebit ex motu centri gravitatis G in plano illo A B D. Atq; ob hujuſmodi ſitum, tale elementum quodvis ſpectari poteſt tanquam punctum Phyſicum p in plano eodem A B D ad punctum z locatum. Reducatur itaq; corpus propoſitum in planum Phyſicum A B D conſtans ex hujuſmodi particulis p.

In hoc plano ut inveniatur punctum O, cujus acceleratio propria non mutatur ab actionibus particularum reliquarum, attendendum eſt ad vires particulæ cujuſvis ſingularis p in puncto z ſitæ. Nam ex hiſce viribus conjunctis oritur plani totius motus abſolutus; cujus ope datur motus puncti cujuſvis propoſiti; unde viciſſim invenitur punctum cujus motus eſt datus.

At urgetur particula p a vi propriæ gravitatis; quæ ſi partium cohæſio diſſolveretur, in dato tempore minimo, datam produceret accelerationem motûs in perpendiculari ad Horizontem z y. Ad C z duc normalem y x, & reſolvetur acceleratio z y in partes z x & x y. Ob corporis rigiditatem, tollitur vis z x per reſiſtentiam puncti C. At vi reliquâ x y trahitur ſpatium A B D in gyrum circa punctum C; & ductâ horizontali C o & perpendiculari z s, erit ea ut $\frac{C\,s}{C\,z}$: Nempe ob gravitatis vim datam, & ſimilia triangula x y z & s C z. Ergo vis particulæ p ad movendum ſpatium ABD eſt ut $\frac{C\,s}{C\,z} \times p$.

Ad has vires in unum colligendas, ſit O punctum invariabile, in lineâ ad libitum ducta & ad diſtantiam adhac incognitam C O. Tum erit vis particulæ p ad movendum

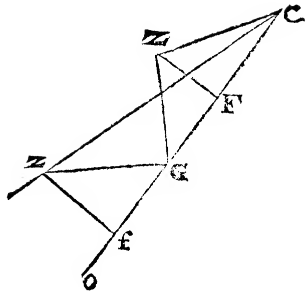
vendum punctum O, ut $\frac{Cz}{CO} \times \frac{Cs}{Cz} \times p$, hoc est ut $\frac{Cs}{CO} \times p$. Acceleratio autem, quam tribuit p eidem puncto O, erit ut $\frac{CO}{Cz} \times \frac{Cs}{Cz}$. Itaq; applicatâ vi illâ $\frac{Cs}{CO} \times p$ ad hanc accelerationem $\frac{CO \times Cs}{Czq:}$, erit quotiens $\frac{Czq:}{COq:} \times p$ particula, quæ, si in ipso puncto O fingatur moveri cum eâdem acceleratione $\frac{CO \times Cs}{Czq:}$, eundem omnino produceret motum, quem in eodem puncto O producit particula p. Hinc demum reducitur Problema ad motuum Theorema notissimum: Applicatâ enim summâ virium $\frac{Cs}{CO} \times p$ ad summam particularum $\frac{Czq:}{COq:} \times p$, erit quotiens acceleratio absoluta puncti O. Dein ductâ perpendiculari O o, & positâ hac acceleratione æquali datæ accelerationi $\frac{Co}{CO}$ ipsius puncti O, dabitur distantia CO. Sit enim $\frac{Co}{CO} = d$, & (juxta methodum *Fluxionum*) $Cs \times p = \dot{M}$, & $Czq: \times p = \dot{C}$. Tum ob CO invariabilem erit summa omnium virium $\frac{Cs}{CO} \times p = \frac{M}{CO}$, & summa omnium particularum $\frac{Czq:}{COq:} \times p = \frac{C}{COq:}$. Unde, applicatâ summâ momentorum ad summam corporum, erit $\frac{M}{C} \times CO = d$ adeoq; $CO = \frac{dC}{M}$. Inventis igitur C & M, per *Fluxionum* methodum inversam, dabitur C O. Q. E. I.

Cor. A centro gravitatis G ad horizontalem C o duc perpendicularem G g, & sit corpus ipsum A B C = A. Tum

Tum ex notissimâ indole centri gravitatis erit $M = Cg \times A$. Unde est $CO = \frac{dC}{Cg \times A}$.

Prop. 2. Theor. 1.

Iisdem positis, quæratur punctum O in rectâ C G transe-



$= CGq: + Gzq: + 2CG \times Gf$. Est ergo $C =$ (aggregato omnium $Czq: \times p =$) aggregato omnium $CGq: \times p + Gzq: \times p - 2CG \times GF \times p + 2CG \times Gf \times p$. At ob centrum gravitatis G, est aggregatum omnium $2CG \times GF \times p =$ aggregato omnium $2CG \times Gf \times p$. Quare est $C =$ aggregato omnium $CGq: \times p + Gzq: \times p = CGq: \times A + D$. At enim *per Theor.*1. est $CO = \frac{C}{CG \times A}$. Ergo $CO = CG + \frac{D}{CG \times A}$. Q. E. D.

Cor. Hinc datur parallelogrammum $CG \times GO$. Est enim $GO = \frac{D}{CG \times A}$. At dantur A & D. Quare datur $CG \times GO = \frac{D}{A}$.

Prop. 4. Theor. 3.

Iisdem positis, si in puncto O constituatur particula physica $\frac{CG \times A}{CO}$, quæ propria gravitate agitata Oscillet circa punctum C; spatij A B C motus perinde omnino erit, ac si agitaretur ab Oscillatione ipsius corporis A.

Constat tam ex Natura centri gravitatis, quam *per Prob* 1. Est enim $\frac{CG \times A}{CO}$ aggregatum omnium $\frac{Czq: \times p}{COq:} = \frac{C}{COq:}$.

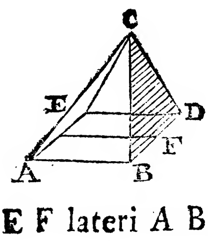
Prop. 5. Prob. 2.

Datis corporis cujuſvis magnitudine A, centro gravitatis G, & puncto ſuſpenſionis C. Invenire ejuſdem centrum Oſcillationis O.

Fit per Theor 1. inveniendo quantitatem C; vel per Theor. 2. quærendo quantitatem D.

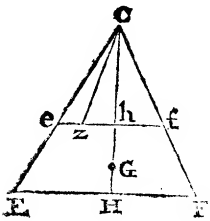
Scholium.

Ad inſtituendum calculum in caſu particulari, eligenda eſt quantitas C vel D, prout ſuggerit natura figuræ propoſitæ. Dein datâ earum alterutrâ, altera item dabitur per æquationem (*Prop.* 3.) $C = CGq: \times A + D$. Unde etiam dabitur pgr. $CG \times GO = \frac{D}{A}$ (*Cor. Prop.* 3.) $= \frac{C}{A} - CGq:$. Cujus ope, ex datis centro gravitatis & puncto ſuſpenſionis, datur centrum Oſcillationis per ſolam diviſionem. Quare in quolibet exemplo ſemper commodiſſimum erit hoc parellelogrammum primùm eruere, vel per computum ipſius D, vel per quantitatem C, ex idoneâ aſſumptione centri ſuſpenſionis.



Supereſt, ut hæc exemplis aliquot illuſtremus.

Ex. 1. Sit figura propoſita Pyramis A D C, cujus baſis eſt pgr. A D, ſitque motus centri gravitatis in plano tranſeunte per verticem C & diametrum baſis E F lateri A B parallelam.



Ad calculum commodiſſime inſtituendum, ſit ipſe vertex C centrum ſuſpenſionis. Tum ad modum *Prob.* 1. reducatur figura ad planum phyſicum trianguli Iſoſcelis C E F, in quo e f parallela ipſi EF repræſentat lineam phyſicam ex particulis p compoſitam. Sit C H = a.

H F

H F = b, & C h = x, Tum ex naturâ figuræ erit e h = $\frac{b\,x}{a}$, & particula p sita ad punctum z erit ut x; vel potius, facto h z = v, erit $\dot{v}\,\dot{x}$ elementi prismatici basis, & p erit ut $\dot{v}\,\dot{x}\,x$. Unde erit $\dot{C} = Czq: \times \dot{v}\,\dot{x}\,x = \dot{v}\,x\,x^3 + \dot{x}\,\dot{v}\,v^2\,x$. Ideoq; summa omnium C z q : × p in lineâ h z erit $v\,\dot{x}\,x^3 + \frac{\dot{x}\,x\,v^3}{3}$; & in lineâ e f (pro v ponendo $\frac{b\,x}{a}$) erit summa illa $\frac{6\,b\,a^2 + 2\,b^3}{3\,a^3} \times \dot{x}\,x^4$. Unde iterum capiendo fluentem, & pro x scribendo a, erit $C = \frac{6\,b\,a^2 + 2\,b^3}{15} \times a^2$. Est autem pyramis ipsa A $= \frac{2\,b\,a\,a}{3}$, & distantia centri gravitatis G a vertice C est C G $= \frac{3}{4}$ a. Unde $\frac{C}{A} - CGq: = \frac{D}{A} = CG \times GO$ $= \frac{3\,a^2 + 16\,b^2}{80}$.

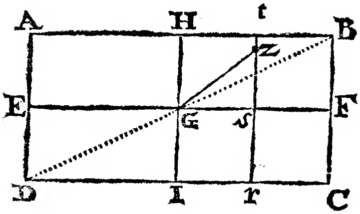
Ex. 2. Sit figura proposita Conus rectus descriptus rotatione trianguli isoscelis E C F circa perpendiculum C H.

Hic iterum sumpto vertice C pro centro suspensionis, & factis C H = a, H E = b, C h = x, h z = v, ut supra; erit $p = 2\,\dot{x}\,\dot{v} \times \sqrt{\frac{b\,b}{a\,a}\,x\,x - v\,v}$; unde $\dot{C} = 2\,\dot{v}\,\dot{x} \times \overline{x\,x + v\,v} \times \sqrt{\frac{b\,b}{a\,a}\,x\,x - v\,v}$. Sit B segmentum circuli diametro e f descripti, quod adjacet Abscissæ h z = v,

& Ordinatæ $\sqrt{\frac{b\,b}{a\,a}x\,x - v\,v}$; tum erit summa omnium $C\,z\,q : \times p$ in rectâ $h\,z = 2\,\dot{x} \times \frac{4\,a^2 + b^2}{4\,a^2}\,x^2\,B - \frac{1}{3}\,\dot{x}\,v \times \overline{\frac{b^2}{a^2}\,x^2 - v^2}\Big|^{\frac{3}{2}}$. Et quando $v = e\,b$, erit hæc summa $2\,\dot{x} \times \frac{4\,a^2 + b^2}{4\,a^2}\,x^2\,B$; cujus duplum $\frac{4\,a^2 + b^2}{a^2}\,\dot{x}\,x^2\,B$ est pars ipsius C in rectâ e f. Est autem area B ut x^2; sit ergo $B = c\,x^2$; atq; pars illa ipsius C erit $\frac{4\,a^2 + b^2}{a^2} \times c\,\dot{x}\,x^4$. Unde capiendo fluentem erit $C = \frac{4\,a^2 + b^2}{5} \times c\,a^3$. Est autem conus ipse $A = \frac{4}{3}\,c\,a^3$, & $C\,G = \frac{3}{4}\,a$. Unde $\frac{C}{A} - C\,G\,q: = \frac{D}{A} = \frac{3\,a^2 + 12\,b^2}{80}$.

Atq; ad hunc modum procedit calculus in alijs figuris, ubi rationes C h ad h e, & h z ad p sunt magis compositæ.

Ex. 3. Ut pateat ratio calculi quantitatis D, sit figura proposita parallelepipedon, cujus facies Horizonti perpendicularis, & parallela plano motûs centri gravitatis est A B D. Duc diametros E F & H I, & sit altitudo elementorum p, : : & sit t r parallela H I; & $G\,F = a$, $G\,H = b$, $G\,s = x$, & $s\,z = v$. Tum erit $\dot{D} = \dot{v}\,\dot{x}\,x\,x + \dot{x}\,\dot{v}\,v\,v$. Unde ipsius D pars in rectâ t r erit $2\,b\,\dot{x}\,x^2 + \frac{2}{3}\,b^3\,\dot{x}$: atq; iterum sumendo fluentis duplum, erit

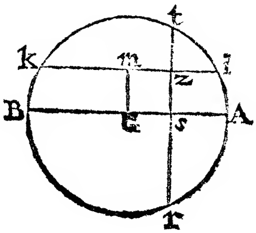


D

$D = \frac{4\,b\,a^3 + 4\,b^3\,a}{3}$. Atqui eſt $A = 4\,a\,b$; unde eſt

$\frac{D}{A} = \frac{a\,a + b\,b}{3} = \frac{1}{12}$ D B quad.

Ex. 4. Sit ultimum exemplum in Sphærâ, cujus circulus maximus B t r, diameter A B, & centrum G. Tum ductis lineis ut in Schemate ſatis patent, erit $\dot{D} = G\,s\,q : \times\, p + G\,m\,q : \times\, p$. At ſumma omnium $G\,s\,q : \times\, p$ in recta t r eſt G s q : ductum in aream circuli diametro t r deſcripti. Item ſumma omnium $G\,M\,q : \times\, p$ in rectâ k i eſt $G\,m\,q : \times$ aream circuli diametro k i deſcripti. Unde ſtatim conſtat eſſe D = quater fluenti ipſius G s q : in aream circuli cujus diameter eſt t r. Sit ergo c area circuli cujus radij quadratum eſt 1, & ſit G A = a, & G s = x. Tum erit $\dot{D} = 4\,\dot{x}\,x\,x \times c\,a\,a - c\,x\,x = 4\,c\,a^2\,\dot{x}\,x^2 - 4\,c\,\dot{x}\,x^4$. Unde ſumendo fluentem, & faciendo x = a, erit $D = \frac{8}{15}\,c\,a^5$. Eſt autem $A = \frac{4}{3}\,c\,a^3$.



Unde $\frac{D}{A} = \frac{2}{5}\,a\,a$.

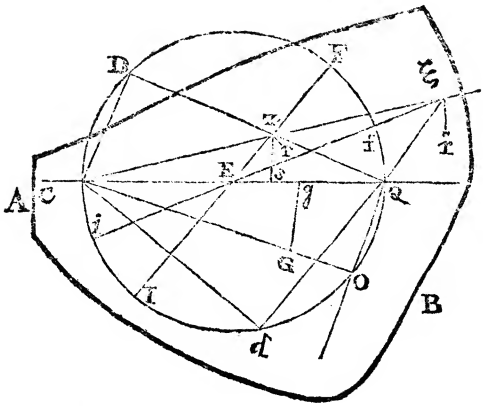
Ob affinitatem ſolutionis libet his ſubjungere Problema de inventione Centri Percuſſionis.

Prop. 6. Prob. 3.

Corporis cujuſvis circa datum punctum rotati, invenire Centrum Percuſſionis; punctum ſcilicet tale, ut Corpus in illud impingens, & eâdem operâ ſolutum a puncto ſuſpenſionis, neque huc neque illuc inclinet.

Primùm conſtat hoc punctum quæri debere in plano motùs centri gravitatis. Si enim corpus reſolvatur in e-

 lementa



lementa prismatica plano isti normalia, ferentur ea motu sibi parallelo; unde momenta ex utrâq; parte istius plani erunt æqualia; adeoq; per resistentiam factam in hoc plano, corporis punctum nullum de eo pelletur. Sit ergo planum illud A B, ad quod reducetur corpus per contractionem elementorum prismaticorum in particulas p ad puncta z sitas, ut in *Prob.* 1. In hoc plano sit C centrum rotationis; aut saltem ejus projectio facta per lineam perpendicularem in hoc planum demissam; & sit Q punctum quæsitum. Per C duc ad libitum C ξ, in quâ sume puncta duo z & ξ, ita ut ductis z Q & ξ Q, sit angulus C z Q obtusus, & angulus C ξ Q acutus: atque in punctis z & ξ sint particulæ p & π. Tum ad C ξ ductis normalibus z r & ξ *r*, quæ sint ad invicem ut C z ad C ξ, ijs repræsentabuntur velocitates absolutæ particularum p & π. At harum velocitatum partes quæ sunt in directionibus z Q & ξ Q, tolluntur per resistentiam puncti Q. Ad Q z & Q ξ duc normales C D & C d, & ob angulos æquales z C D = r z Q, & ξ C d = *r* ξ Q, velocitatum partes reliquæ, in directionibus ipsis Q z & Q ξ perpendicularibus, erunt ut z D & ξ d. Unde habitâ ratione distantiarum Q z & Q ξ, erunt vires particularum p & π ad movendum spatium A B in partes contrarias, ut D z × z Q × p, & d ξ × ξ Q × p. At per conditiones Problematis debent summæ hujusmodi contrariarum virium esse inter se æquales.

Ob

Ob angulos ad D & d rectos, sunt puncta D & d ad circumferentiam circuli diametro C Q descripti. Sit istius circuli centrum E. Tum ductis E z & E ξ circulo occurrentibus in F & I, f & i, erit $Dz \times zQ = Fz \times zI = EFq: - Ezq: = EQq: - Ezq:$, & $d\xi \times \xi Q = E\xi q: - EQq:$ Quare erit summa omnium $EQq: \times p - Ezq: \times p =$ summæ omnium $E\xi q: \times \pi - EQq: \times \pi$; & terminis transpositis, summa omnium $EQq: \times \overline{p + \pi}$: = summæ omnium $Ezq: \times p + E\xi q: \times \pi$, hoc est, si p ponatur tam pro particulâ p intra circulum, quam pro particula π extra circulum, erit summa omnium $EQq: \times p =$ summæ omnium $Ezq: \times p$. Ad C Q duc normalem z s. Tum erit $Ezq: = Czq: + ECq: - QC \times Cs$. Quo valore ipsius $Ezq:$ ei substituto, & æquatione debité tractatâ, tandem invenies summam omnium $CQ \times Cs \times p =$ summæ omnium $Czq: \times p$. Unde est $CQ = \frac{\text{summæ omnium } Czq: \times p}{\text{summ: omnium } Cs \times p}$. At enim est summa omnium $Czq: \times p$ ipsa quantitas C in calculo centri Oscillationis: & si centrum gravitatis sit G, & ad C Q ducatur normalis G g, & corpus ipsum dicatur A, erit summa omnium $Cs \times p = Cg \times A$. Unde est $CQ = \frac{C}{Cg \times A}$. Sit centrum Oscillationis O; tum per *Theor.* 1. erit $CO = \frac{C}{CG \times A}$. Unde est $Cg : CG :: CO : CQ$. Quare per O ducta ad C O perpendicularis transibit per punctum Q. Q. E. I.

III. *Epistola*